Fundamental Astronomy and h- Geometry

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Section 1.
There are quite a few books on astronomy. We will use [1].
Classic.
We have a system of trigonometric equations [1] p.19, (2.16)

$$\sin(A)\cdot\cos(a) = \sin(hm)\cdot\cos(\delta)$$

$$\cos(A)\cdot\cos(a) = \cos(hm)\cdot\cos(\delta)\cdot\sin(\phi) - \sin(\delta)\cdot\cos(\phi)$$

$$\sin(a) = \cos(hm)\cdot\cos(\delta)\cdot\cos(\phi) + \sin(\delta)\cdot\sin(\phi) \quad (1)$$

We know hm δ, ϕ. Need to find A ,a. Let's face it

$$D1 = \sin(hm)\cdot\cos(\delta)$$

$$D2 = \cos(hm)\cdot\cos(\delta)\cdot\sin(\phi) - \sin(\delta)\cdot\cos(\phi)$$

$$D3 = \cos(hm)\cdot\cos(\delta)\cdot\cos(\phi) + \sin(\delta)\cdot\sin(\phi)$$

$$a = \operatorname{asin}(D3) \quad (2)$$

$$D4 = \frac{D1}{\cos(a)}$$

$$A = \operatorname{asin}(D4) \quad (3)$$

To calculate a and A, it will be necessary to count trigonometric functions 8 times. As you know, this can only be counted by infinite lines

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots \quad (4)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots. \quad (5)$$

$$\operatorname{asin}(x) = \sum_{n=0}^{\infty}\left[\frac{(2\cdot n)!}{4^n\cdot(n!)^2\cdot(2\cdot n+1)}\cdot x^{2\cdot n+1}\right] \quad (6)$$

As we will see again, despite all the advancements in computer development, this causes some problems. In the beginning, how many line members need to be evaluated? And clearly, where computing time is very limited, the issue of computing time remains relevant. Note that in classical trigonometry, common initial numbers are given in degrees, minutes and seconds. All of this needs to be converted to radians, because in function calculation formulas, angles must be given in radians. Once the results of the calculation are obtained (in radians), conversion to degrees is required.

h - Geometry model.
We will give expressions for h - Geometry functions

$$sph = \frac{h}{\sqrt{h^2 + (1-h)^2}} = z \qquad h = \frac{z}{z + \sqrt{1-z^2}} \tag{7}$$

$$cph = \frac{1-h}{\sqrt{h^2 + (1-h)^2}} = z \qquad h = 1 - \frac{z^2 - z\cdot\sqrt{1-z^2}}{2\cdot z^2 - 1} \tag{8}$$

$$tph = \frac{h}{1-h} = z \qquad h = \frac{z}{1+z} \tag{9}$$

The following formulas are used when comparing computational results based on geometry

$$\alpha = \operatorname{atan}\left(\frac{h}{1-h}\right) \tag{10}$$

$$h = \frac{\tan(\alpha)}{1+\tan(\alpha)} \tag{11}$$

Let's move from degrees to h for parameters. Then we can write instead of (1)

$$sp(hA)\cdot cp(ha) = sp(hhm)\cdot cp(h\delta)$$

$$cp(hA)\cdot cp(ha) = cp(hhm)\cdot cp(h\delta)\cdot sp(h\phi) - sp(h\delta)\cdot cp(h\phi)$$

$$sp(ha) = cp(hhm)\cdot cp(h\delta)\cdot cp(h\phi) + sp(h\delta)\cdot sp(h\phi) \tag{12}$$

Adding sph and cph values and rearranging expressions will yield

$$a = \frac{1}{1 + \sqrt{\frac{1}{M^2} - 1}} \tag{13}$$

Where

$$M = \frac{1}{L\delta\cdot L\phi}\cdot\left(\frac{Hhm}{Lhm}\cdot H\delta\cdot H\phi + h\delta\cdot h\phi\right)$$

$$L\delta = \sqrt{h\delta^2 + (1-h\delta)^2} \qquad Lhm = \sqrt{hhm^2 + (1-hhm)^2} \qquad L\phi = \sqrt{h\phi^2 + (1-h\phi)^2}$$

$$H\delta = 1 - h\delta \qquad Hhm = 1 - hhm \qquad H\phi = 1 - h\phi$$

Knowing the values of hδ, hhm and hφ will only require one algebraic expression to be counted (13).
Adding sph and cph values and rearranging expressions will yield

$$A = \frac{1}{1 + \sqrt{\frac{1}{N^2} - 1}} \qquad (14)$$

Where

$$N = \frac{hm \cdot Hh\delta \cdot Lha}{Lhm \cdot Lh\delta \cdot Hha}$$

Specific calculations.
A calculation example is taken from [1] p.42 Example 2.4 Here accepted

$$hm = 51.08^{o} \qquad \delta = 14.70^{o} \qquad \phi = 60.16^{o} \qquad (15)$$

Classic.
To calculate the values of sin and cos, the angles given must be converted to radians and then using (4), (5) we find the values of sin and cos. Then using formulas (2), (3), (6), we will calculate a and A.

$$a = 0.549756 \qquad A = 1.081381 \qquad (16)$$

What matches the results in the book [1]

h - Geometry model.

The model h - based on geometry is (12). Using (11) the angles measured in degrees (radians) will be converted to the angles measured in h parameters. We will

$$hm = 0.553258 \quad h\delta = 0.207824 \quad ha = 0.379949 \qquad (17)$$

We need to find a and A. Using algebraic expression (13) we get

$$a = 0.549756 \qquad (18)$$

Using the algebraic expression (14) we get

$$A = 1.081381 \qquad (19)$$

As we can see, the calculations as in the classical trigonometry method (16), so using the mathematical models of h-geometry (18), (19) are completely identical.
But there is a fundamental difference:

- For calculations using classical trigonometry models, where angles are measured, calculations based on infinite rows must be used at least nine times.
- Calculating on the basis of h - geometry it is enough to calculate two algebraic expressions.

Literature
1. Fundamental Astronomy Hunnu Kurttunen, Pekka Kroger, Heikki Oja, Markku Poutanen, KarlJ.Donner, Fifth Edition Springer. 2006